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Time and Events

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Time plays a special role in Standard Quantum Theory. The concept of time observable causes many controversies there. In Event-Enhanced Quantum Theory (EEQT) Schrödinger's *differential equation* is replaced by a *piecewise deterministic algorithm* that describes also the *timing of events*. This allows us to revisit the problem of time of arrival in quantum theory.

1. INTRODUCTION

Event-Enhanced Quantum Theory (EEQT) was invented to answer John Bell's concerns about the status of the measurement problem in quantum theory (Bell, 1989, 1990). EEQT's main thesis is best summarized in the following statement:

NOT ALL IS QUANTUM

Indeed, a pure quantum world would be dead. There would be no events; nothing would ever happen. There is no dynamics in pure quantum theory to explain how potentialities become actualities. And we do know that the world is not dead. We know events do happen, and they do it in finite time. This means that pure quantum theory is inadequate. John Bell realized this fact and at first he sought a solution in hidden variable theories (Bell, 1987a). Rudolf Haag (Haag, 1990a,b, 1995, 1996) takes a similar position; he calls it an "evolutionary picture." EEQT is motivated by the same concerns, but takes a slightly different perspective. What EEQT has in common with hidden variable theories [as well as Bell's (1987b) "beables"] is the realization that

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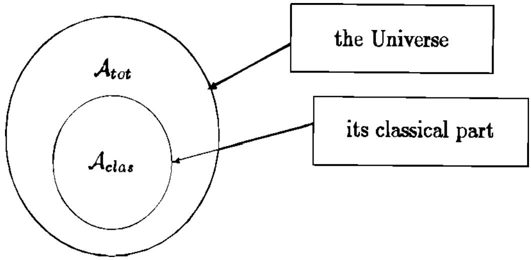

Fig. 1.

THERE IS A CLASSICAL PART OF THE UNIVERSE

and this part can evolve. "We" (IGUS-es) belong partly to this classical world. Once the existence of the classical part is accepted, then *events* can be defined as changes of state of this classical part (Fig. 1). EEQT is the only theory that we are aware of that can precisely define the two concepts

- Experiment
- Measurement

thus complying with the demands set by Bell (1989, 1990). We define (Jadczyk, 1995)

- *Experiment*: completely positive one-parameter family of maps of $\mathcal{A}_{\text{tot}}$
- *Measurement*: very special kind of experiment

Moreover, EEQT is the only theory where there is one-to-one correspondence between a linear Liouville equation for ensembles and an individual *algorithm* for generation of events. It is to be noted that, although EEQT does not involve hidden variables, it does seek for deeper than just statistical descriptions. Namely, it asks the following question:

- *How* does nature create the unique world around us and our own unique perceptions?

In other words, EEQT seeks knowledge by going beyond the pure descriptive orthodox interpretation.[3] In agreement with Rudolf Haag, we take an evolutionary view of nature. This means that the future does not yet exist; it is being continuously created, and this creation is marked by events. But *how* does this process of creation proceed? This is what we want to know. A hundred years ago the answer would have been: "by solving differential equations." But today, after taking lessons in relativity and quantum theory, after the computer metaphor has permeated so many areas of our lives, we

[3] We would like to quote at this place this, simple, but deep, wisdom; *Knowledge protects, ignorance endangers.*

propose to seek an answer to the question of "how" in terms of an *algorithm.* Thus we set up the hypothesis: *Nature uses a certain algorithm that is yet to be discovered.* Quantum theory tells us clearly: this algorithm is nonlocal. Relativity adds to this: nonlocal in space implies nonlocal in time. Thus we have to be prepared to meet acausalities in the individual chains of events even if they average out in large statistical samples. EEQT can be thought of as one step in this direction. It proposes its piecewise deterministic process [PDP of Blanchard and Jadczyk (1995a,b)] as the algorithm for generating a sample history of an individual system. This algorithm should be thought of as a fundamental one, more basic than the master equation which follows from it after taking a statistical average over different possible individual histories. In EEQT it still holds that there is one-to-one correspondence between PDP and the master equation, and it is easy to think of an evident and unavoidable generalization of PDP, when feedback is included, which will go beyond the linear master equation and thus beyond linear quantum theory. Work in this direction is in progress.

According to the philosophy of EEQT, the quantum state vector is an auxiliary variable which is not directly observable, even in part. It is a kind of a hidden variable. But, according to EEQT, there *are* directly observable quantities—and they form the $\mathcal{A}_{clas}$ part of $\mathcal{A}_{tot}$. EEQT does not assume standard quantum mechanical postulates about results of measurements and their probabilities. All must be derived from the dissipative experiment dynamics by observing the events at the classical level, i.e., by carring out continuous observation of the state of $\mathcal{A}_{class}$.

An *event* is thus a fundamental concept in EEQT and there are two primitive event characteristics that the algorithm for event generation must provide: the "when" and the "which," and indeed PDP is a piecewise deterministic algorithm with *two* "roulette" runs for generating each particular event. First the roulette is run to generate the *time of event.* Only then, after the timing has been decided, is there a second roulette run that decides, according to the probabilities of the moment, *which* of the possible events is selected to occur. Then, once these two choices have been made, the selected event happens and is accompanied by an appropriate quantum jump of the wave function. After that, continuous evolution of possibilities starts again, roulette wheels are set into motion, and the countdown begins for the next event.

2. EVENT-GENERATING ALGORITHM

No event can ever happen unless a given quantum system is coupled to a classical system. In fact, the reader should be warned here that this statement is not even precise. A precise statement would be: "no event can happen to a system unless it contains a classical subsystem." In many cases, however, the total system can be considered a direct product of a pure quantum system and a classical one. If we restrict ourselves to such a case, then the simplest nontrivial event generator is a "fuzzy property detector" defined as follows. Let Q be a pure quantum system whose (uncoupled) dynamics is described by a self-adjoint Hamiltonian H acting on a Hilbert space $\mathcal{H}$. A fuzzy property detector is then characterized by a positive operator F acting on $\mathcal{H}$. In the limit of a "sharp" property we would have $F^2 = \sqrt{\kappa} F$, where κ is a numerical coupling constant (of physical dimension t^{-1}). That is the property becomes sharp for F proportional to an orthogonal projection.

According to a general theory described in Blanchard and Jadczyk (1995a, b), a property detector is a two-state classical device, with states denoted 0 and 1 and characterized by the transition operators [using the notation of Blanchard and Jadczyk (1995a, b)]:$g_{01} = 0$, $g_{10} = F$. The master equation describing continuous time evolution of statistical states of the quantum system coupled to the detector reads

$$\dot{\rho}_0(t) = -i[H_0, \rho_0(t)] + F\rho_1 F$$
$$\dot{\rho}_1(t) = -i[H_1, \rho_1] - \tfrac{1}{2}\{F^2, \rho_1\} \tag{1}$$

Suppose at $t = 0$ the detector is off, that is, in the state denoted by 0, and the particle state is $\psi(0)$, with $\|\psi(0)\| = 1$. Then, according to the event-generating algorithm described heuristically in the previous section, the probability $P(t)$ of detection, that is, of a change of state of the classical device, during time interval $(0, t)$ is equal to $1 - \|K(t)\,\psi(0)\|^2$, where $K(t) = \exp(-iH_0 t - \Lambda t/2)$, where $\Lambda = F^2$. It then follows that the probability that the detector will be triggered in the time interval $(t, t + dt)$, provided it was not triggered yet, is $p(t)\,dt$, where $p(t)$ is given by

$$p(t) = \frac{d}{dt} P(t) = \langle K(t)\psi_0, \Lambda K(t)\psi_0\rangle \tag{2}$$

Let us consider the case of a maximally sharp measurement. In this case we would take $\Lambda = |a\rangle\langle a|$, where $|a\rangle$ is some Hilbert space vector. It is not assumed to be normalized; in fact its norm stands for the strength of the coupling (note that $\langle a|a\rangle$ must have physical dimension t^{-1}). From this formula it can be easily shown (Blanchard and Jadczyk, 1996) that $p(t) = |\phi(t)|^2$, where the Laplace transform $\tilde{\phi}(z)$ of the (complex) amplitude $\phi(t)$ is given by the formula

$$\tilde{\phi} = \frac{2\langle a|\tilde{K}_0|\psi_0\rangle}{2 + \langle a|\tilde{K}_0|a\rangle} \tag{3}$$

where $K_0(t) = \exp(-iH_0 t)$.

3. TIME OF ARRIVAL

Let us consider a particular case of *time of arrival* [see Muga *et al.* (1995), for a recent discussion]. Thus we take $|a\rangle$ to denote a position eigenstate localized at the point a, that is, $\langle x|a\rangle = \sqrt{\kappa}\delta(x - a)$, κ being a coupling constant representing efficiency of the detector. For the Laplace transform $\tilde{\phi}$ of the probability amplitude we obtain then

$$\tilde{\phi} = \frac{2\sqrt{\kappa}}{2 + \kappa\tilde{K}_0(a, a)}\,\tilde{\psi}_0(a) \tag{4}$$

where $\tilde{\psi}_0$ stands for the Laplace transform of $K_0(t)\psi_0$.

Let us now specialize to the case of free Schrödinger particle on a line. We study the response of the point counter to a Gaussian wave packet whose initial shape at $t = 0$ is given by

$$\psi_0(x) = \frac{1}{(2\pi)^{1/4}\eta^{1/2}} \exp\left(\frac{-(x - x_0)^2}{4\eta^2} + 2ik(x - x_0)\right) \tag{5}$$

In the following it will be convenient to use dimensionless variables for measuring space, time, and the strength of the coupling:

$$\xi = \frac{x}{2\eta}, \qquad \tau = \frac{\hbar t}{2m\eta^2}, \qquad \alpha = \frac{m\eta\kappa}{\hbar} \tag{6}$$

We denote

$$\xi_0 = x_0/2\eta, \qquad \xi_a = a/2\eta, \qquad v = 2\eta k \tag{7}$$

$$u_\pm = i\sqrt{-iz} \pm (v - id), \qquad d = \xi_0 - \xi_a \tag{8}$$

The amplitude $\tilde{\phi}$ of equation (4), when rendered dimensionless, then reads

$$\tilde{\phi}(z) = (2\pi)^{1/4}\alpha^{1/2}e^{-d^2-2ivd}\,\frac{w(u_+) + w(u_-)}{2\sqrt{iz} + \alpha} \tag{9}$$

with the function $w(u)$ defined by

$$w(u) = e^{-u^2}\,\mathrm{erfc}(-iu) \tag{10}$$

The time-of-arrival probability curves of the counter for several values of the coupling constant are shown in Fig. 2. The incoming wave packet starts at $t = 0$, $x = -4$, with velocity $v = 4$. It is seen from the plot that the average time at which the counter placed at $x = 0$ is triggered is about one time unit, independent of the value of the coupling constant. This numerical example shows that our model of a counter can be used for measurements

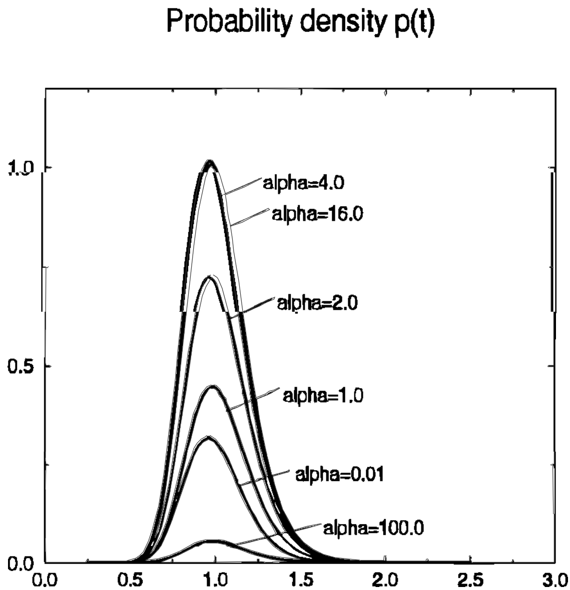

Fig. 2. Probability density of time of arrival for a Dirac delta counter placed at $x = 0$, with different values of coupling constant alpha. The incoming wave packet starts at $t = 0$, $x = -4$ with velocity $v = 4$.

of time of arrival. It is to be noticed that the shape of the response curve is almost insensitive to the value of the coupling constant. It is also important to notice that in general the probability $P(\infty) = \int_0^\infty p(\tau)d\tau$ that the particle will be detected at all is less than 1. In fact, for a pointlike counter as above, the numerical maximum is <0.73 (Blanchard and Jadczyk, 1996). For this reason (i.e., because of the need of normalization) the *time-of-arrival observable is not represented by a linear operator.*

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